INTRODUCTION

Chronic kidney disease (CKD) is a significant public health issue. In the 2015 Global Burden of Disease Study, overall CKD mortality has increased by 11.7% over the last 20 years, making it one of the fastest rising major causes of death [4]. As a result, the number of people in need of a kidney transplant is growing rapidly, and being on the wait list is one of the few possibilities to receive a transplant. Unfortunately, there are not enough deceased kidney donations to help everyone in need [1, 2].

In Canada alone, over 1,600 people are added to organ transplant wait lists every year. Roughly 44.3% of patients on dialysis treatments survive at least five years.

Kidney paired donation (KPD) programs aim to increase living donor kidney transplantation as an alternative to the traditional deceased donor transplantation.

KIDNEY PAIRED DONATION (KPD)

When a living kidney donor is incompatible with the recipient, they are considered an incompatible patient-donor pair. A paired kidney exchange occurs when recipients swap their donors in order to get a compatible kidney. Potential donors and recipients can be incompatible due to blood-type mismatch and the presence of antibodies threatening the new organ transplants. Exchanges can have either the form of cycles or chains. Cycles give rise to the extended edge (EE) integer program [3], which we improve through the addition of new constraints and variables to obtain optimal solutions quickly. Although the KEP is a hard-to-solve problem, advanced mathematical techniques can be applied to obtain optimal solutions quickly. We tested our formulation for several values of \(k\) and compared it with the existing extended edge formulation (EE) [3].

The bounded and extended edge formulation (BEE) of KEP is as follows:

Maximum \( \sum_{(i,j) \in E_L} w_{ij} x_{ij} \) subject to

- \( \sum_{(i,j) \in E_L} x_{ij} = 1 \) for each directed edge \((i,j)\) present in the graph
- \( \sum_{(i,\ldots,j') \in E_L} w_{ij} x_{ij} \leq 1 \) for each directed edge \((i,j')\) in the graph
- \( \sum_{(i,j') \in E_L} w_{ij} x_{ij} \leq k \) for each directed edge \((i,j')\) in the graph
- \( w_{ij} \geq 0 \) for each directed edge \((i,j)\) in the graph
- \( x_{ij} \in \{0, 1\} \) for each directed edge \((i,j)\) in the graph

As observed in Figure 4, our model (BEE) is very consistent and effective to find the optimal solution in much less time than the EE formulation and always far below the time limit.

To test the mathematical formulations, we generate 32 random instances of sizes between 10 and 100 patients. We use the same definitions and constraints to find the optimal solutions quickly.

CONCLUSION

- Kidney paired donation is a well-established alternative to increase living donor kidney transplantation.
- Although the KEP is a hard-to-solve problem, advanced mathematical techniques can be applied to obtain optimal solutions quickly.
- Our approach was effective for the real-sized instances tested, outperforming another well-known integer programming formulation.

REFERENCES